Total mixed domination number of a graph

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Abstract

A subset $S \subseteq V \cup E$ of a graph G is a total mixed dominating set of G if each element of $V \cup E$ is either adjacent or incident to an element of S. The total mixed domination number $\gamma_{tm}(G)$ of G is the minimum cardinality of a total mixed dominating set. Here, we talk on this concept which is initiated its studying in [1].

1 Some Results

Some of our results are listed in following. Recall that $\gamma_t(G)$ and $L(G)$ denote the total domination number and the line graph of G, respectively.

Theorem 1. *For any connected graph G with $\delta(G) \geq 1$,*

$$\max\{\gamma_t(G), \gamma_t(L(G))\} \leq \gamma_{tm}(G) \leq \gamma_t(L(G)) + \gamma_t(G).$$

Theorem 2. *For any coonected graph G, let n_L be the nuumber of disjoint paths of lentgh 3 between two connected components of the induced subgraph of the line graph $L(G)$ by a minimum total dominating set of $L(G)$, and let n_G be the number of disjoint paths of lentgh 3 between the connected components of the induced subgraph of G by a minimum total dominating set of G. Then*

$$\gamma_{tm}(G) \leq \min\{\gamma_t(L(G)) + n_L, \gamma_t(G) + n_G\}.$$

Corollay 1. *Let $L(G)$ be the line graph of a graph G which has a minimum total dominating set such that the induced subgraph of $L(G)$ by it is connected. Then $\gamma_{tm}(G) = \gamma_t(L(G))$.*

Theorem 3. *For any connected graph G of order at least 2, if diameter of G is at least 4, then $\gamma_{tm}(G) \geq 3$.*

Theorem 4. *For any graph G of order $n \geq 2$ which has a Hamiltonian path,*

$$\gamma_{tm}(G) \leq \begin{cases} \lfloor 2n/3 \rfloor & \text{if } n \equiv 0 \pmod 3, \\ \lfloor 2n/3 \rfloor + 1 & \text{if } n \equiv 1, 2 \pmod 3. \end{cases}$$

Theorem 5. *For any tree $\mathbb{T}$ of order $n \geq 3$, $\gamma_{tm}(\mathbb{T}) \leq 2n/3$.*

The total mixed domination number of some known graphs are given in the following propositions.

Proposition 1. *For any path P_n of order $n \geq 2$,*

$$\gamma_{tm}(P_n) = \begin{cases} 4\lceil n/7 \rceil - 3 & \text{if } n \equiv 1 \pmod 7, \\ 4\lceil n/7 \rceil - 2 & \text{if } n \equiv 2, 3, 4 \pmod 7, \\ 4\lceil n/7 \rceil - 1 & \text{if } n \equiv 5 \pmod 7, \\ 4\lceil n/7 \rceil & \text{if } n \equiv 0, 6 \pmod 7. \end{cases}$$

Proposition 2. *For any cycle C_n of order $n \geq 3$,*

$$\gamma_{tm}(C_n) = \begin{cases} 4\lceil n/7 \rceil - 3 & \text{if } n \equiv 1 \pmod 7, \\ 4\lceil n/7 \rceil - 2 & \text{if } n \equiv 2, 3 \pmod 7, \\ 4\lceil n/7 \rceil - 1 & \text{if } n \equiv 4 \pmod 7, \\ 4\lceil n/7 \rceil & \text{if } n \equiv 0, 5, 6 \pmod 7. \end{cases}$$

Proposition 3. *For any integers $n \geq m \geq 1$, let $K_{m,n}$ be a complete bipartite graph. Then*

$$\gamma_{tm}(K_{m,n}) = m + 1.$$

Proposition 4. *For any wheel W_n of order $n + 1 \geq 4$, $\gamma_{tm}(W_n) = \lceil n/2 \rceil + 1$.*

References

[1] A. P. Kazemi, F. Kazemnejad. Total mixed domination in graphs. Manuscript.