Cut-Covering Decompositions for Connectivity Problems

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Abstract

We study decompositions of graphs that cover small-cardinality cuts an even number of times, and we use these tools to design approximation algorithms for the traveling salesman problem (TSP) and the 2-edge-connected spanning multigraph problem (2EC) on restricted classes of weighted graphs. For example, Boyd, Iwata and Takazawa gave an algorithm to find a cycle cover that covers all 3- and 4-edge cuts in a bridgeless, cubic graph. We apply this to connectivity problems on *node-weighted*, 3-edge-connected, cubic graphs. To extend this approach to 2-edge-connected graphs, we present a procedure to decompose a solution for the subtour linear program into spanning, connected subgraphs that cover each 2-edge cut an even number of times, and we apply this to 2EC on subcubic, node-weighted graphs. Finally, we apply these decomposition tools to the problem of *uniform covers*, answering a question of Sebő.

This talk will mainly present the work from [1].

Many algorithms for the traveling salesman problem (TSP) and the 2-edge-connected spanning multigraph problem (2EC) rely on some form of graph decomposition. For example, Wolsey's analysis of Christofides' famous algorithm uses the fact that an optimal solution for the subtour linear program dominates a convex combination of spanning trees, and this same LP solution scaled by half dominates a convex combination of T-joins for any spanning tree. Another combinatorial approach for TSP on a graph metric (graph-TSP) is to find Eulerian subgraphs whose union spans the vertex set while containing few edges and few connected components, e.g. a cycle cover with few cycles.

We study decompositions of graphs that cover small-cardinality cuts an even number of times, and we use these decompositions to design algorithms with improved approximation guarantees for the traveling salesman problem (TSP) and the 2-edge-connected spanning multigraph problem (2EC) on restricted classes of weighted graphs. For example, Boyd, Iwata and Takazawa recently presented an algorithm to find a cycle cover that covers all 3- and 4-edge cuts in a bridgeless, cubic graph with applications to 2EC on 3-edge-connected, cubic graphs [2]. We present several applications of their algorithm to connectivity problems on *node-weighted* graphs, which has been suggested as an intermediate step between graph and general metrics. Specifically, on 3-edge-connected, cubic, node-weighted graphs, we present a $\frac{7}{5}$-approximation algorithm for TSP and a $\frac{13}{10}$-approximation algorithm for 2EC.

To extend this approach to graphs that are 2-edge-connected, we present a procedure to decompose an optimal solution for the subtour linear program into spanning, connected subgraphs that cover each 2-edge cut an even number of times. We use this decomposition to design a simple $\frac{4}{3}$-approximation algorithm for 2EC on subcubic, node-weighted graphs.

Finally, motivated by the well known "four-thirds conjecture", we apply these decomposition tools to the problem of *uniform covers*. For a cubic, 3-edge-connected graph, we show that the everywhere $\frac{18}{19}$ vector can be efficiently written as a convex combination of tours, answering a question of Sebő [3]. Additionally, for such graphs, we show that the everywhere $\frac{15}{17}$ can be efficiently written as a convex combination of 2-edge-connected spanning multigraphs.

References

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[3] Y. Benchetrit, A. Sebő, M.Stehlík. Problems about uniform covers, with tours and detours. Oberwolfach Report. 2014.