10th International Colloquium on Graph Theory and Combinatorics, July 9-13, 2018

ICGT 2018, France, Lyon

Graphs whose size of all maximal 2-regular subgraphs are the same

Somayeh Khalashi Ghezelahmad — Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran
S. Akbari — Sharif University of Technology

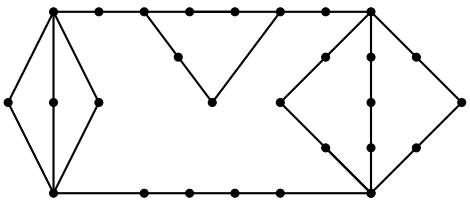
Abstract

In this paper, we study graphs whose size of all maximal 2-regular subgraphs are the same. We call these graphs equi-2-regular. We characterize regular graphs which are equi-2-regular. We study claw-free equi-2-regular graphs. Moreover, a family of 2-connected equi-2-regular graphs is constructed.

1 Introduction

All graphs we consider are finite, simple and undirected. Let G be a graph. The minimum degree of G is denoted by $\delta(G)$. A graph G is called *equimatchable* if all maximal matchings in G, have the same size. For example $K_{n,n}$, where n is a positive integer, is an equimatchable graph. The concept of equimatchability was first considered by Meng [8], Lewin [5] and Grünbaum [2] in 1974. Since then, equimatchable graphs have been extensively studied by several authors. See for instance [1, 3, 4, 6]. Sumner [6], proved that a connected equimatchable graph has a 2-factor if and only if it is one of the graphs K_{2n} and $K_{n,n}$, where n is a positive integer. Favaron also proved that every equimatchable factor-critical 2-connected graph is Hamiltonian. Equimatchable 3-regular graphs are characterized in [3]. It is proved that the only connected 3-regular equimatchable graphs are K_4 or $K_{3,3}$.

By a maximal 2-regular subgraph, we mean a 2-regular subgraph which is not included in a 2-regular subgraph of bigger size. In this paper, we intend to characterize all graphs whose size of all maximal 2-regular subgraphs are the same We call these graphs *equi-2-regular*. For instance, the following graph is equi-2-regular and the size of its maximal 2-regular subgraphs is 18.



We study regular graphs which are equi-2-regular. We investigate claw-free equi-2-regular graphs. Furthermore a family of 2-connected equi-2-regular graphs is constructed.

2 Main Results

Theorem 1. *Let r be a positive integer and G be an r-regular graph which is not a cycle. Then G is not equi-2-regular.*

Lemma 1. *If G has a cycle with chord such that the chord induces K_3, then G is not equi-2-regular.*

Proof. To see this, let C_1 be a cycle with chord. Now, there are cycles $C_2, \ldots, C_k$, $k \geq 2$ such that $C_1 \cup C_2 \cup \cdots \cup C_k$ is a maximal 2-regular subgraph of G. For each $i = 1, \ldots, k$, let t_i be the size of C_i. (Note that for $i \geq 2$, t_i can be 0). Let $e = uv$ be the chord of C_1 and the path $Q = u, w, v$ be the part of C_1 such that u, w, v induces K_3. Let P be the u, v-path on C_1 which is disjoit from Q. Now, if $(P \cup e) \cup C_2 \cup \cdots \cup C_k$ is a maximal 2-regular subgraph, then clearly its size is different from $t_1 + \cdots + t_k$. If $(P \cup e) \cup C_2 \cup \cdots \cup C_k$ is not maximal, then we can extend it by adding a cycle C_{k+1} passing through w. Now $(P \cup e) \cup C_2 \cup \cdots \cup C_k \cup C_{k+1}$ is a maximal 2-regular subgraph with size $(t_1 - 1) + \cdots + t_k + t_{k+1} \geq t_1 + \cdots + t_k + 2$. It follows that G is not equi-2-regular. □

Theorem 2. *Let G be a connected claw-free graph which is not a cycle. If $\delta(G) \geq 3$, then G is not equi-2-regular.*

Proof. Let C be a cycle of G of length at least 4. Since G is connected and is not a cycle there is a vertex $z \in V(G) \setminus V(C)$ such that z is adjacent to some vertex $v \in V(C)$. Let u, v, w be three consecutive vertices of C. Now, since G is claw-free either u is adjacent to w or z is adjacent to u or w. In each case, G has a cycle with chord such that the chord iduces K_3. Thus by Lemma 1, we are done. □

References

[1] O. Favaron, Equimatchable factor-critical graphs, J. Graph Theory 10 (1986) 439-448.

[2] B. Grünbaum, Matchings in polytopal graphs, Networks, 4 (1974), 175–190.

[3] K. Kawarabayashi, M. D. Plummer, and A. Saito, On two equimatchable graph classes. Discrete Math., 266 (2003), 263–274.

[4] M. Lesk, M. D. Plummer, and W. R. Pulleyblank, Equi-matchable graphs, Graph Theory and Combinatorics, Proc. Cambridge Combinatorial Conference in Honour of Paul Erdos. B. Bollob´as, (editor), Academic Press, London (1984), pp. 239–254.

[5] M. Lewin, Matching-perfect and cover-perfect graphs. Israel J. Math., 18 (1974), 345–347.

[6] D. P. Sumner, Randomly matchable graphs, J. Graph Theory 3 (1979) 183-186.

[7] D.B. West, Introduction to Graph Theory, Second Edition, Prentice Hall, Inc, 2001.

[8] D. H.-C. Meng, Matchings and coverings for graphs. PhD thesis, Michigan State University, East Lansing, MI, 1974.