The Eccentricity Connectivity Index of Certain Edge-Gluing Graphs

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Abstract

The eccentric connectivity index denoted by ε^c is a novel distance-based molecular structure descriptor. It is defined as $\varepsilon^c(G) = \sum_{v \in V(G)} ecc\ v\ dv$ where *ecc v* eccentricity of v and dv is degree of vertex v. In this paper, we computed the eccentricity connectivity index of certain graphs such as cycle and complete graphs and then computed their edge-gluing graphs.

Keywords: Eccentricity Connectivity Index, Cycle, Complete Graphs, Edge Gluing

1 INTRODUCTION

A graph G is a tuple *G(V,E)*, which consists of a finite nonempty set *V(G)* of elements called vertices, and a finite set *E(G)* of unordered pairs of distinct elements of *(G)* . The topological indices of graph G are graph-theoretical invariants designed to find relationships between the structure of chemical molecules and their physical properties. One of these index is the eccentric connectivity index introduced in 1997 by Sharma et. al. [10]. The eccentric connectivity index has been found to have a low degeneracy and hence a significant potential of predicting biological activity of certain classes of chemical compounds [9]. In 2010 Bo Zhou and Zhibin Du determine the n-vertex trees of diameter with the minimum eccentric connectivity index [2]. In 2014 Jayenthi A. and Kulandai Therese present the explicit formula for the values of eccentric connectivity index for some special graphs [3]. In 2014 Roslan Hasni et. al. computed eccentricity connectivity polynomials of some families of dendrimers [7]. In 2017 P. Padmapriya and Veena Mathad establish formula to calculate the eccentric-distance sum for some graphs [6]. In the present work, we investigate the eccentricity connectivity index of some special graphs, as well as obtained edge gluing of these graphs.

2 BASIC DEFINITIONS

2.1 Definition [5]

The distance d_{uv}, between two vertices u and v is the shortest length joining them.

2.2 Definition [5]

The eccentricity of a vertex u, *ecc u*, in a connected graph is the maximum distance between u and v , where $v \in V(G)$, i.e $ecc\ u = max_{(v \in V(G))}\ d_{uv}$.

2.3 Definition [5]

The diameter of a graph G denoted by *d(G)*, is the maximum eccentricity in G, i.e $d(G) = max\ ecc\ u = max_{(v \in V(G))}\ d_{uv}$.

2.4 Definition [5]

The radius of a graph G denoted by $r(G)$, is the minimum eccentricity among all vertices u in G: $r(G) = min\ ecc\ u = min_{(v \in V(G))}\ d_{uv}$.

2.5 Definition [4]

A cycle graph is one that is obtained by joining the two end-vertices of a path graph. Thus, the degree of each vertex of a cycle graph is two. A cycle graph with n vertices denoted by C_n,(Fig.1).

2.6 Definition [5]

A complete graph is the graph with any two vertices are adjacent. The number of edges in a complete graph is $n(n\text{-}1)/2$. We denote the complete graph on n vertices as K_n. (Fig. 2).

2.7 Lemma

(i) The eccentricity connectivity index of a cycle graph C_n is $\varepsilon^c\ (C_n) = 2n\lfloor n/2 \rfloor$.
(ii) The eccentricity connectivity index of a complete graph K_n is $\varepsilon^c\ (K_n) = n(n\text{-}1)$

2.8 Definition [8]

Let G_n and G_m be graphs, each one of them containing a complete sub-graph k_p ($p \geq 1$). If G is, the graph obtained from the union of G_n and G_m by fusing the two sub-graphs k_p in any arbitrary way (Fig. 3) , then we call G a k_p -gluing of G_n and G_m. and denoted by G_n^m.

2.9 Definition [1]

The e-gluing of cycle graph is a graph obtained from two different cycle graphs C_n and C_m with one common edge e_g denoted by C_n^m, where g_1=(fusion v_1 with u_1) and g_2 = (fusion v_2 with u_2) (Fig. 4).

2.10 Remarks

(i) The diameter d of graph C_n^m is $d = \lfloor (n + m - 2)/2 \rfloor$.
(ii) The radius r of graph C_n^m is $r = max \lfloor m/2 \rfloor, \lfloor n/2 \rfloor$.

2.11 Definition

The e - gluing complete graph is a graph obtained from two different complete graphs k_n and k_m with one common edge e_g denoted by k_n^m, where g_1=(fusion v_1 with u_1) and g_2 = (fusion v_2 with u_2) (Fig. 5).

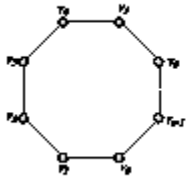


Fig 1. Cycle Graph C_n.

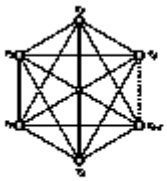


Fig 2. Complete Graph K_n

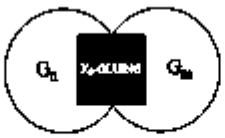


Fig 3 Gluing Graph G_n^m

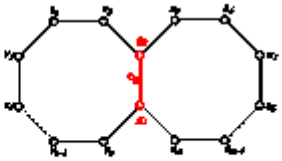


Fig 4. e-gluing Cycle Graph C_n^m

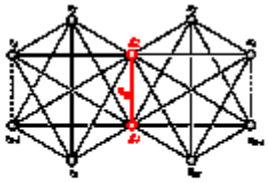


Fig 5. e-gluing Complete Graph K_n^m

3 THE MAIN RUSELETS

Theorem 1. *Let n,m and i be a positive integers, d and r are diameter and radius of C_n^m respectively, m, $n \geq 3$. Then the eccentricity connectivity index of e-gluing of cycle graph C_n^m is:*
$\varepsilon^c(C_n^m) =$

$$\begin{cases} 4d^2+4r^2-2r-8dr+2nr+2mr & if \quad n,m \quad odd \quad and \quad m+n-2 \geq 6 \\ 4d^2+4r^2+4d-6r-8dr+2nr+2mr & if \quad n,m \quad even \\ 4d^2+4r^2+6d-8r-8dr+2nr+2mr & if \quad one \quad odd \quad and \quad the \quad other \quad even, n, m \geq 4 \\ 2mr+4r & if \quad n=3, \quad m \quad and \quad m \geq 6 \\ 14 & if \quad n,m=3 \\ 24 & if \quad n=3, m=4 \end{cases}$$

Proof: If n,m odd and $m+n-2 \geq 6$. There are$(n+m-2)$ vertices in C_n^m. Among which two vertices of degree 3 with eccentricity r, two vertices of degree with eccentricity d, $[n+m-6-4(d-r-1)]$ vertices of degree 2 with eccentricity r and four vertices of degree 2 with eccentricity i (where $r+1 \leq i \leq d-1$) . Then by using definition of eccentricity connectivity index we have

$$\varepsilon^c(C_n^m) = 2[3r]+(2)[2d]+[n+m-6-4(d-r-1)][2r]+4[2(r+1)]+\cdots+4[2(r-2)]+4[2(r-1)] \quad (1)$$

$$\varepsilon^c(C_n^m) = 6r+4d+[n+m-6-4(d-r-1)](2r)+8\sum_{i=r+1}^{d-1} i = 4d^2+4r^2-2r-8dr+2nr+2mr \quad (2)$$

If n,m even . There are (n+m-2) vertices in C_n^m Among which two vertices of degree 3 with eccentricity r , four vertices of degree 2 with eccentricity d, [n+m-8-4(d-r-1)] vertices of degree 2 with eccentricity r and four vertices of degree 2 with eccentricity i (where $r+1 \leq i \leq d-1$). Then by using definition of eccentricity connectivity index we have

$$\varepsilon^c(C_n^m) = 2[3r]+(2)[2d]+[n+m-8-4(d-r-1)]2r+4[2(r+1)]+\cdots+4[2(r-2)]+4[2(r-1)] \quad (3)$$

$$\varepsilon^c(C_n^m) = 8r^2+8d-8dr-2r+2nr+2mr+8\sum_{i=r+1}^{d-1} i = 4d^2+4r^2+6d-8r-8dr+2nr+2mr \quad (4)$$

If n=3 ,m even and m≥6. There are (n+m-2) vertices in C_3^m. Among which two vertices of degree 3 with eccentricity r and (m-1) vertices of degree 2 with eccentricity r. Then by using definition of eccentricity connectivity index we have

$$\varepsilon^c(C_3^m) = 2(3r)+(m-1)2r = 4r+2mr \quad (5)$$

If n=m=3. There are 4 vertices in C_3^3. Among which two vertices of degree 3 with eccentricity 1 and two vertices of degree 2 with eccentricity 2. Then by using definition of eccentricity connectivity index we have $\varepsilon^c(C_3^3) = 2[3(1)] + (2)[2(2)] = 14$. If n=3, m=4. There are 5 vertices in C_3^4. Among which two vertices of degree 3 with eccentricity 2 and three vertices of degree 2 with eccentricity 2. Then by using definition of eccentricity connectivity index we have $\varepsilon^c(C_3^4) = 2[3(2)]+(3)[2(2)] = 24$.

Theorem 2. *Let n and m be a positive integers, then the eccentricity connectivity index of $e-$ gluing of complete graph K_n^m is*

$$\varepsilon^c(K_n^m) = 2(n^2 + m^2 - 2n - 2m + 1) \tag{6}$$

Proof: There are (n+m-2) vertices in K_n^m. There are (n+m-2) vertices in C_n^m. Among which two vertices of degree (n+m-3) with eccentricity 1, (n-2) vertices of degree (n-1) with eccentricity 2 and (m-2) vertices of degree (m-1) with eccentricity 2. Then by using definition of eccentricity connectivity index we have

$$\varepsilon^c(K_n^m) = 2[(n+m-3)1] + (n-2)[(n-1)2] + (m-2)[(m-1)2] \tag{7}$$

$$\varepsilon^c(K_n^m) = 2(n^2 + m^2 - 2n - 2m + 1) \tag{8}$$

4 CONCLUSION

In this paper, we give new graphs obtained from gluing certain graphs (cycle and complete) by one edge then give their eccentricity connectivity index.

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