On Stronger Types of Locating-dominating Codes

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Abstract

We present some results about γ^{SLD} and γ^{DLD}, the parameters associated to self-locating-dominating codes and solid-locating-dominating codes in graphs. Both stronger types of locating-dominating codes where recently introduced following the ideas of self-identifying codes. We study these parameters in special graph families, such are trees and Cartesian product of graphs. We also present results about the relationship between each of them and the well-known locating dominating number.

A sensor network can be modelled as a finite, simple, connected and undirected graph $G = (V, E)$ as follows: the set of vertices V of the graph represents the locations of the network and the edge set E of the graph represents the connections between the locations. So sensors can be placed in selected vertices of the graph and the sensor placed in the vertex c monitors c itself and the vertices neighbouring c. A non-empty subset C of V is called a *code* and the elements of the code are called *codewords*. In our context, the code C represents the set of locations where the sensors have been placed on. For the set of sensors monitoring a vertex $u \in V$, we use the notation $I(C; u) = N[u] \cap C$.

A sensor $c \in V$ reports that an irregularity has been detected if there is (at least) one in the closed neighbourhood $N[c]$. Notice that if the sensors in the code C are located in such places that $I(C; u)$ is nonempty and unique for all $u \in V$, then an irregularity in the network can be located by comparing $I(C; u)$ to identifying sets of other vertices. This leads to the well-known definition of identifying codes [3].

Definition 1. A code $C \subseteq V$ is a identifying code in G if for all distinct $u, v \in V$ we have $I(C; u) \neq \emptyset$ and $I(C; u) \neq I(C; v)$.

Self-identifying codes where introduced in [1] as a stronger form of identifying codes, which are able to locate one irregularity on a network as well as detect multiple ones.

Definition 2. A code $C \subseteq V$ is a self-identifying code in G if it is identifying in G and for all $u \in V$ and $U \subseteq V$ such that $|U| \geq 2$ we have $I(C; u) \neq I(C; U)$.

The following two characterizations can be found in [1] for self-identifying codes.

Theorem 3. *Let C be a code in G. Then the following statements are equivalent:*

1. *The code C is self-identifying in G.*
2. *For all distinct $u, v \in V$, we have $I(C; u) \setminus I(C; v) \neq \emptyset$.*
3. *For all $u \in V$, we have $I(C; u) \neq \emptyset$ and $\bigcap_{c \in I(C;u)} N[c] = \{u\}$.*

In case that just irregularities in non-codewords are of interest, the appropriate property to be considered is location-domination [4].

Definition 4. A code $C \subseteq V$ is locating-dominating in G if for all distinct $u, v \in V \setminus C$ we have $I(C; u) \neq \emptyset$ and $I(C; u) \neq I(C; v)$.

Comparing the definitions of identifying and locating-dominating codes, we immediately notice their apparent similarities; in the case of identification we require that the identifying sets $I(u)$ are unique for all vertices and in the case of location-domination the same is required for non-codewords. Therefore, as self-identifying codes are a natural specialization of regular identifying codes, it is obvious to consider if something similar could be done for locating-dominating codes. Indeed, the characterizations of Theorem 3 give two natural ways to define new types of locating-dominating codes with similar kind of beneficial properties as self-identifying codes have over regular identifying codes. The definitions of these codes are given as follows [2].

Definition 5. 1. A code $C \subseteq V$ is self-locating-dominating in G if for all $u \in V \setminus C$ we have $I(C;u) \neq \emptyset$ and $\bigcap_{c \in I(C;u)} N[c] = \{u\}$.

2. A code $C \subseteq V$ is solid-locating-dominating in G if for all distinct $u, v \in V \setminus C$ we have $I(C;u) \setminus I(C;v) \neq \emptyset$.

It is shown in [2] that every self-locating-dominating code in G is also solid-locating-dominating in G, and that every solid-locating-dominating code in G is also locating-dominating in G. However they are not equivalent properties.

In the same way that the number of codewords in a minimum locating-dominating code is called the *location-domination number* and is denoted by $\gamma^{LD}(G)$, the parameters associated to both new definitions are the following.

Definition 6. 1. The self-locating-dominating number of G is the cardinal of a minimum self-locating-dominating code and it is denoted by $\gamma^{SLD}(G)$.

2. The solid-locating-dominating number of G is the cardinal of a minimum solid-locating-dominating code and it is denoted by $\gamma^{DLD}(G)$.

We present some results about these parameters, such are general upper bounds and graphs attaining them and the behaviour of both numbers in trees and in Cartesian product of graphs.

We also present two results that explore the relationship between the pairs of parameters $\gamma^{LD}(G), \gamma^{SLD}(G)$ and $\gamma^{LD}(G), \gamma^{DLD}(G)$, by characterizing which values of them can be simultaneously achieved in a graph.

References

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